## **IMO 2023 Student Report**

## Gabriel Goh (SGP4)

#### **Abstract**

This is written on 26th August, mostly from memory. Please message me if you would like me to remove any details due to privacy concerns, same goes if I remembered something wrongly. I've tried to give the contestant number of anybody I mention here. Certain details (arguably the best ones) are omitted because I'm not spoiling X-men night. On the topic of spoiling, reading this will spoil the IMO wide open (duh!). This is entirely in sans serif cos it's the superior font. On a more important note, thank you to Xingyu (POR6) for inspiring me to do this.

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## 1 Introduction

IMO 2023 was held in Chiba, Japan from the 2nd to 13th of July (The contestant programme started on the 6th). This marked the second time the IMO was held IRL since the pandemic. The Singapore delegation comprised:

- The 6 contestants: Wenhao (SGP1), Drew (SGP2), Edden (SGP3), Gabriel (SGP4), Yan Xi (SGP5), Qinxi (SGP6). 4 of the contestants also participated in IMO '22 in Oslo.
- 5 trainers: Aloysius, Wee Kean, Xu Chen, Shevonne, David. The first 3 were observer Bs, while Shevonne was also an observer B in all but name. David was sadly unable to join us in Japan.
- Mr Zong (Deputy Leader), Dr Soh (Leader), Dr Tay (Observer A).

Our guide was Nicholas (from Malaysia).



Figure 1: (From left) Bus, Me, Yanxi, Xu Chen, Edden, Aloysius, Dr Tay, Wee Kean, Wenhao, Nicholas, Qinxi, Mr Zong, Dr Soh, Drew, Shevonne.

#### 2 Pre-IMO

#### 2.1 NTST

Selection was done over two weekends (4 4.5h papers) in April. Questions were taken from the 2022 ISL, and in order of appearance:

A2 C3 G5 / G3 N3 A5 / C1 G4 N5 / N1 A4 C7.

The cutoff was 56/84 (i.e. exactly 8 questions). There were definitely better question choices (and some questionable choices) but what has been done has been done (and hey, isn't it the same story every year). Tiebreaks are done by APMO scores (and funnily enough, the fact that APMO results were released after IMO results this year became a major meme at the IMO).

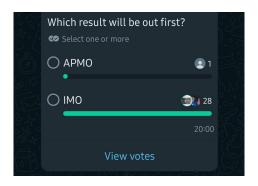


Figure 2: One of many messages making fun of APMO on the mass IMO contestant whatsapp chat.

#### 2.2 Intensive Training

The last week of May was reserved for intensive (non stay-in) training, followed by SIMO Camp in the first few days of June. There was a 15 day stay-in training "camp" in June, which was pretty much us rotating between doing math and going crazy, and occasionally both concurrently. (It is easy to go insane staring at equations, trust me.) This was followed by another week (T3W1) of non stay-in training, and then a break starting from Singapore MO R2 on July 1st.

Rather notably, we had a combined mock with the Iranian and Taiwanese teams, with questions set by trainers - I thought that was a cool concept. Aloysius scared us with the "none of yall will solve my q6" (he was right) while Taiwan haunted us (me) with some cursed geom q5. Interestingly, the questions are now under "International Contests" on AoPS.



Figure 3: Last day of training at NUS!

#### **3** IMO

All times are approximations. All places are approximations. All details are approximations. I did not expect this to be so detailed...

#### 3.1 July 6: Arrival

(July 5, 20:00) Our plane flight from Changi Airport was unfortunately at midnight, around 2am. Some of the team went to Jewel to eat dinner but I managed to fall asleep from 8pm to 10pm so I only met the rest of them at the designated reporting time of 10:49 (I wonder why). Basically everyone was there, along with everyone's parents (and siblings). Edden boasted that one-third of his luggage was empty he was going on a shopping spree. We took some photos and then checked in.

(00:00) Not much happened inside - The 5 of us (excluding Edden) decided to just walk around the airport and took photos in front of a sign that said "A1,A2,A3,A4,A5,A6,A7,A8".



Figure 4: Which would come out at the IMO?

By the time we were at the departure lounge I think everyone was tired, the observers played a few games of 99, and the 6 of us tried to guess our contestant numbers (which, irritatingly, we never knew until then).

(02:00) Expectedly, nothing much happened on a 2am to 10am flight besides sleeping. The inflight meal wasn't particularly great either. Yanxi and I laughed about how the safety video spent a good 30 seconds explaining how to slide down the emergency slide, in the same way one would to a toddler who has never went down a slide at a waterpark before. We also tried exploring this weird thing known as "in-flight exercises" which was basically a code word for professional fidgeting. In the end we just all slept for a good 5 hours. Qinxi later told me that he couldn't sleep and spent some time working on an FE (18A1, I think).

(10:30) We cleared immigration without much hassle and met two of the German contestants at the luggage area. When we exited we were told to board a bus, which was beautifully empty. The guide told us that if nobody came on by 11am the bus would leave (Japanese punctuality!) and of course we were really happy when that happened. Drew and I spent most of it staring out of the window (Drew was investigating the car plate system, I was just daydreaming) while most of the team were at the very

back presumably playing some game.

(12:00) We reached the front desk pretty quickly, with a couple of minor traffic jams. Roughly 30 seconds after we entered the building somehow someone managed to scream "who is gghx" at us, which we quickly recognised as the Australian team. Somehow gghx precedes me?? Drew and I had a good time talking to Chris (AUS4). The two of them are among the most active members on MODS and I guess its always pretty awkward when you meet, for the first time in a year, someone you've had thousands and thousands of messages with. Pretty common theme over the next few days XD.

(13:00) At this point we also met our guide Nicholas, whom we first thought was Singaporean due to his name. Turns out he was from Malaysia so we weren't that far off! He is a university student in Tokyo (also first time in Chiba) and brought us out for lunch at some nearby mall. The restaurant had an iPad to take orders, and my table had fun using the camera to take photos.



Figure 5: First meal in Japan.

It also happened to be really hot in Japan so I decided to prematurely change into the SIMO'23 shirt (which we had to wear on the opening ceremony). In fact, a few days after we left, Japan was faced with yet another extreme heatwave. I doubt it rained a single time on the 8 days I was there.

Somehow, the Switzerland team managed to end up at the same restaurant as us, just one table behind, and that went unnoticed by us until I received the following message from Tanish (SUI DL) on Instagram.

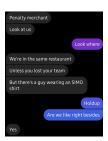


Figure 6: Message from Tanish.

Bora **(SUI1)** came over and convinced me that the Swiss team was doing a video project where they filmed contestants wearing the Swiss hat and convinced me to take a photo in it - which I obliged - but as it turns out he simply had a 100 Yen bet with Tanish that he could make someone do it. I also met Tanish for the first time (follow @swiss.mo on instagram for MO memes) and he later became a major source of entertainment at the IMO (thanks!).

(14:30) When we returned to the front desk we were informed that our room keys would only be given out at 4, in alphabetical order, which in effect meant that we waited until 5+. We passed the time playing Hanabi and taking some video for guy for his Instagram. (We resorted to smashing a bell with our gift pandas and shouting "Skill Issue".) We were issued our bags (really good quality, but hey it's Japan after all) which contained a really nice cap, and some foolscap. There was also a shirt, which were different colours for contestants, observers, and DL.



Figure 7: Some merch.

(17:00) We finally got our keys, honestly the check-in process was pretty atrocious, it was just a hap-hazard mess of hundreds of people crowding in the middle of the lobby scrambling to take things. I met Orestis (HEL2) and bumped into Bora again, this time he was attempting to solve some India TST Q3. He asked me if I wanted to try it but having done so much math in the past month I wasn't in the mood. Yanxi took a slight interest (it was geom after all) but I guess he too was not in the mood. Somewhere around this point I realised there were two lounges dedicated to the IMO, named the "Crane" and "Dragon" rooms. I made a mental note to visit this later.

(17:30) We went into our hotel rooms (on the 10th floor of a massive 50 storey building). The hotel came with basically everything, EXCEPT that the rooms were tiny. Wenhao and I basically worked on a space optimisation layout for a while (ok, maybe it wasn't that hard, we just shoved everything to one side) and then we went for dinner at the hall.



Figure 8: Our "restaurant" for the next 7 days.

(18:00) The dining hall was a pretty spectacular sight but the food was absymal. I was expecting at the very least Japanese food, but the only thing they had was Japanese rice. We were, however, quite amused at the small packets of margarine: the proper way to open it was to take the two ends and squeeze, after which a steady flow will form in the middle. Unnecessarily cool Japanese design! At this point the Indian team had arrived, and I had a good time talking to Atul (IND6), Siddarth (IND2) and Archit (IND4). Once again, a bunch of people I've talked to so often (GC, anybody?) but have never met.

The Swiss team brought souvenirs (one of them, I believe Felix, gave me the Swiss cap, which I happily wore to all the excursions). Tanish decided to advertise a book featuring Swiss universities with lines such as "Everything is free except for the tuition and accommodation!" and pointing out random numbers in the book. The book was eventually passed around everywhere and even ended up with one of the Jane Street Hub organisers (I totally didn't tell them it was lost and found).



Figure 9: This book went through at least 20 countries.

I also met Celestia (THA6) (I can't imagine myself calling him by any other name) as well as Rafael

(IDN1). Edden had a good time trashtalking the NUS courses (Rafael is studying there now). Going into IMO I gave myself a side quest of exchanging as many currencies as possible, inspired by the Costa Rican and Honduras team last year. This proved way more difficult than I thought, for the simple reason that people don't bring their currency around, and my social skills are severely lacking. Central and South Asia turned out to be a really easy region to collect, and I had a good chat with the Pakistan team. I was slightly worried I wouldn't be able to hit my 30 currency target (I brought 30 SGD 1 dollar coins).

(20:00) I had no idea what the rest of Singapore was doing at this point, they were probably in their rooms or something. I went to the Dragon room first, and was met by free-flow snacks and some arcade machines.



Figure 10: Free Food!!!



Figure 11: The Jane Street Dragon room.

There were also a bunch of tables where people were playing cards or folding origami, and an air hockey table that was perpetually crowded. I met the Taiwanese team and we discussed (I summoned what little

knowledge of Chinese I had left) about the Iran-Singapore-Taiwan questions. I asked them if this was going to be a good year (medal wise) and I think I accidentally made them more stressed LOL (but they did well though). We then spent some time talking about the ISL (especially the hilariously misplaced C6) and I asked them about their selection tests (in particular, the squid game question, which I had became pretty invested in). I then spent the next 5 minutes farming currencies, until I was given a lecture on the Bosnian convertible mark and I decided I had initiated enough conversations with strangers for the day.

I went over to the Crane room, which was basically a board game fiesta. The Indian team was playing 5v1 sets, against their DL who had apparently written his master thesis on the game, making it a suprisingly fair fight. Atul and I agreed to play a game of chess, which was a pretty standard 1700-ish game - I was pretty sure I played a losing move when I decided to blow the position open, but he didn't see the line (or maybe I calculated wrongly). The indian team later came around to spectate and Archit decided to adopt the role of trash-talker (for anyone curious about the game - it evolved into a pawn endgame and then a queen endgame, and we ended up drawing, except that Bora appeared and told us a random *zwischenzug* near the very end was winning for Atul). The room closed at 9 and I remember telling Atul "good luck for the papers!" to which he replied "wait it's not tomorrow right?".

(21:30) I bathed and went over to Drew and Edden's room, where we discussed a little about contest strategy and random overpowered theorems, after which some Hong Kong contestants came over and I left them to play card games in their room. I think we all slept well - the beds were extremely comfy to make up for the lack of room space (to be fair, we never needed the non-bed parts of our rooms anyways).

#### 3.2 July 7: Opening Ceremony

We woke up and went for breakfast (which was equally absymal) and then left for a walk to a park new the hotel. As stereotype goes, there wasn't a single dustbin anywhere on the streets (in fact, that became a frequent problem where the only "dustbin" we could find was the one in the hotel lobby's toilet). July 7 happens to be a festival in Japan, known as *Tanabata*, or the Star Festival. It is a tradition to write well-wishes on small pieces of paper and hang them off trees, and we were amused to find quite a few relating to the IMO.



Figure 12: Singapore writing some well wishes on a tree in a park?

(10:00) At some point I decided to buy food to feed the fish in the park, and managed to drop the entire plastic wrapping into the pond. Xu Chen volunteered to climb over and retrieve it (thanks!). He later joked that I had used up my feeding capacity for the week, and would not feed during the IMO (how true, take notes guys! (Placebo effect anyone?)). We then went to visit this giant flower clock, which was well, a flower clock.



Figure 13: It's a functioning clock btw.

And then, as is typical of every math olympiad team, we spent a good half an hour at a playground and mini zipline. Wee kean and I bet on who could last longer jumping on a trampoline - I think it was a really good demonstration of the maturity of the average MO contestant.



Figure 14: Playgrounds are the best!

On the way back, we stocked up on snacks to eat during the contest. Wenhao told me that Kit Kat was a popular gift for exam-takers, because it sounded like something in Japanese which probably meant something good. I kinda forgot but I bought it anyways. I went with Wenhao, Yanxi and Aloysius to go explore the hotel, and the top few levels were pretty much closed except Level 46, the sponsor booths,

which offered an amazing view of the coastline and the city.



Figure 15: Print-out photo by Huawei of SGP+HUN.

We took a photo with the Hungarian team, and somehow we decided to ask their Deputy Leader if he set Komal questions, and the answer was surprisingly yes! (I later learnt that Komal was part of the Hungary's selection process.) Aloysius remarked that the Hungarian contestants looked pretty stressed, and we left them soon afterwards.



Figure 16: View from level 46!

(13:30) The opening ceremony only started at 3, but Japan being Japan we were basically all seated by 2. We took some pictures outside the Hall and were given giant placards by country. I met Kaixin (CAN3) and we joked about his year long quest to solve a geom problem (he got a 1/7 a week ago, and was understandably proud of it). Inside, the Philippines team in front of us passed the time by solving the Spiral Galaxy on the back of our shirts, while the Thai team behind us passed the time by being characteristically noisy. There were a bunch of questionable ads playing that predictably got made fun of at MODS.



Figure 17: Ad started with a father dying and ending with life insurance. It's always life insurance companies isn't it?

I then offloaded a set of keychains (which Mr Zong gave us to give out) to the New Zealand team... in short it was a pretty boring hour.

(15:00) Country leaders arrived around 15 minutes prior to much fanfare, we tried to attract the attention of Dr Soh and I think he waved at us? The ceremony proper was pretty entertaining and featured a (presumably) traditional Japanese drum kickoff, followed by some band performances. There was also a list of boring speeches which I thankfully remember nothing of. Countries then went up separately - Colombia had an elaborate formation, Portugal hit us with another siuuu, Tanish wore a shirt saying Shaqiri>>>Pessi and there was thunderous applause for Ukraine. Drew and I later found out that Alexander (USA5) spent the ENTIRE TIME recording the duration of each country on the stage, and live updated on MODS, which is some real dedication.



Figure 18: As per tradition we brought Simon up.

(18:00) Through the opening ceremony, I kept exchanging currency, and hit about 15 by the end of the day. As we left, we took photos with the Indonesian team.



Figure 19: Singapore flag (extended edition).

I managed to find the Belarus team and asked them for some cash, and for some inexplicable reason they were soo suspicious of me I almost died under the embarrassment. Dinner was once again pretty uneventful and the Singapore team retreated to the observer's room (which was unfairly large) to discuss contest strategy for a good hour. I don't remember anything much but it was definitely helpful (although tbf my strategy was just go in and get as many marks as I can, which is not so much a strategy as the blatantly obvious thing to do). I went back and glanced through my notes, learnt how to spell nullstellensatz, and then went to sleep. Once again the sleep was fantastic, thank God for the beds!



Figure 20: Front view and back view of the things I brought into the examination hall, we were allowed to put what we wanted into an A4 sized ziplock bag, on my bed. I packed it the night before.

## 3.3 July 8: Day 1

(07:30) Our meals were provided in "bento" boxes, and having no appetite for anything most of us just didn't eat much (what a waste!).



Figure 21: Breakfast!

We walked over to Makuhari Messe, and because its Japan we were made to enter at 8:15. Rather

irritatingly, there was completely no concept of time in the room, nobody knew how close it was to anything. There wasn't a clock, and only a giant "4:30" countdown at the front. As Drew later told me, he wasn't used to thinking downwards, and honestly a clock couldn't be that hard to include? The table was expectedly large and the 5 cards were the same as last year's. Having arrived so early and being so bored all the contestants started walking aimlessly around the hall. We had a mini GC meetup in the middle and I probably disturbed Chris multiple times. On several occasions, a group of students would suddenly start clapping, and being the immature kids we were, the entire hall will erupt in rapturous applause, much to the confusion of the Japanese organisers (in fact, this too happened during dinner yesterday). At some point I realised I didn't bring a ruler in (!), and luckily I borrowed one from Drew, who came in with 3 30cm rulers, phew!

(13:30) All in all it was a pretty smooth day, the question favoured me, I was done pretty early and started spamming toilet. The organisers took roughly forever to collect the scripts, and I was extremely hungry at the end (despite consuming like a week's allowance of chocolate). I've always wondered how many calories are burnt in a 4.5 hour paper (especially in IMO setting), and if classical chess is any basis of comparison to go by, I would say a lot. As I exited the hall I met Edden, who basically said the equivalent of "I told you this would happen", a reference to his fear of a medium geom (in my head I was thinking "surely you solved it??" but of course I just nodded). By the time we left it was already past 2 (of course, having no sense of time, I didn't know this until I asked Aloysius) and we hurried back into our hotel rooms to eat lunch - Japanese food - which tasted amazing.

(15:00) Singapore has a practice of individually reporting progress to the observers, Wenhao went first and took a solid 20 minutes. While he was gone I decided to reinstall Discord and check AoPS, which I previously decided not to check - an FC changes things. Everything was fine, my solution mirrored that of the first solution posts, and I learnt that Ivan Chan was the proposer for Q3. I had a chat with him which roughly involved me telling him 5 times it was a really good problem (it was!) (but maybe not for q3). Expectedly I see Kaixin trolling on the Q2 thread, poor guy must be devastated at the thought of double geom.

I went for "interrogation" after Wenhao, which was a pretty straightforward affair - Xu Chen looked impressed at my proof for increasing, and to be fair I was impressed to. I joked with Aloysius that Q3 was a poly FE (the FE part wasn't so relevant, but it definitely helped - proving increasing is a standard idea in FEs), a reference to his Iran-Singapore-Taiwan Q6 (go check it out!). I also remember sketching a Q2 diagram with two parallel lines clearly intersecting at 45 degrees, which was pretty funny. (Wee Kean later told me that I looked quite sad after exiting the exam venue, I think that was a result of being bored for the first time in a competition.)

(16:00) After that was done, I spent a good hour chilling in the bathtub, experimenting with wave harmonics (not for the sake of H2 physics, it just turns out that investigating wave harmonics in a stationary body of water is a surprisingly natural thing to do). Wenhao and I later tried to draw the Q2 diagram on the mirror of the toilet using the condensation - we were bored.



Figure 22: Wasn't pretty bad for a freehand.

Dinner was a pretty muted affair, there were still contestants coordinating with their deputy leaders but nobody was in the mood for anything much. We then went back for another round of contest strategy discussion, and Mr Zong announced that all of us managed to solve the Q2 geom, which seemed to lift everyone's spirits. That entire day Wenhao and I spoke absolutely *nothing* about our performances, although I was pretty confident he FCed too (based on his past performances and lack of despair). Mr Zong would later be impressed about this, although I personally thought (and still think) it was the logical and easy thing to do.

#### 3.4 July 9: Day 2

(07:30) The process was the same as yesterday, we had bento boxes which were mostly left unfinished. The examination hall was even crazier this time, and some genius decided to initiate *races* at the back of the hall, roughly a 150m shuttle run, which I am devastated nobody has a record of. This paper (Q5) was a lot harder, and quite a bit more painful. I struck gold (oh wow, accidental pun) with 90 minutes left, when I looked back at one of the first pieces of rough work and the DP materialised. I don't think my mood has shifted from two extremes so fast before. The collection process was equally cumbersome and I spent the time speaking to the guy behind me (I can't remember the country), who rather impressively managed to solve Q5 but not Q4. After the paper I spoke with Rickson (PHI4) who solved Q5 but not Q2... Was Q5 supposed to be easy?? Luckily, my sanity was restored when I went to the toilet: everybody was complaining about Q5. On my way out I met with Qinxi who told me he didn't solve anything - he was understandably disappointed.

(2:30) I learnt that Wenhao didn't solve Q5 back in our room, which was quite wasted. Lunch was a pretty similar affair, we ate in our rooms and then went for another round of interrogating. I checked AoPS again and was happy to see my partial progress on the Q6 thread. I spent a good deal of time talking to Drew and Lance (NLD1) about the problems and the cutoffs, and found out that Drew solved Q5 in 1 hour, and thus found it easier than Q2 as well. (I maintained, and still maintain, that Q5 is harder than Q3). How we spent so long outside our room talking about 6 problems still remain a mystery, but at this point I also found out how Edden did (he was expecting 763770 and was slightly irritated about messing up Q3) as well as Yanxi (I asked "did you solve Q6", which in hindsight was the worst possible thing to ask). My coordination went quite smoothly (I already told the observers I solved Q5 with partial on Q6, so they understandably didn't really need much detail).

(17:00) Nicholas brought us (the 5 of us, excluding Qinxi) to the beach. In fact, Wenhao left early, so we met him there. Unlike the beaches I'm used to, the water level remained surprisingly shallow for a far distance.



Figure 23: In the middle of nowhere.

We spent a while playing with the water and enjoying the breeze (Edden lost his specs inside, it's a miracle he found it), and then proceeded to try to balance logs in the sea, which proved to be much harder than we thought (ok tbf, it was obviously really hard, we were just idealists).



Figure 24: It definitely did not fall in the next 10 seconds.

Nicholas cooked up a game of "you have 30 seconds to find any object, the one who can throw it furthest wins" and the only thing I learnt from that is that throwing sticks like a javelin is a waste of time. We then used the logs and sticks to draw a Q2 diagram in the sand. After about an hour, Nicholas made us sit on a log and meditate, which I found pretty calming.



Figure 25: The three logs are obviously concurrent.

(19:00) We had another quick uneventful dinner, and I decided to go with Qinxi and Yanxi to explore the Jane Street Hubs, but before that Qinxi pulled up his fake Windows bluescreen onto the display computer, which hilariously was still there on the next day. In the Dragon room we found a... (the next 30 minutes are too good to be left in a report).



Figure 26: I absolve myself of all responsibility wrt this image.

(21:00) Around this point Dylan and Zhao Yu arrived at our hotel; they had volunteered to be coordinators. After giving us some snacks and introducing us to 5 and a half, Zhao Yu complained about the 100 P1 scripts that he had to grade. The observers were busy preparing for coordination and I just continued camping in Drew's room.

#### 3.5 Contest Analysis

No report is complete without some discussion about the paper itself, so here goes:

To quote UK Leader Dominic Yeo, "in my subjective opinion it was potentially the best set of questions for (at least) ten years". I won't go so far, but definitely the best paper by far since 2020. There was a clear progression of difficulty on both days, topic distribution was normal, a medium problem on the harder side and a hard problem on the easier side, leading to good differentiation (unlike the disaster that was 2022). It turns out that Q1, Q5, Q6 were obvious choices during the jury meeting, which I guess was not much of a surprise.

Cutoff predictions this year were pretty accurate, it was generally agreed that 17/18 for Bronze, 23/24 for Silver and 33/34 for Gold was about there. I think the accuracy of predictions reflects a well-set paper: the differentiators are clear enough that the ranges are pretty much decided up to  $\pm 1$ .

#### 3.5.1 Q1

Straightforward, small cases gives the solution set and with "prime powers" there is nothing else to try except "consider two primes". I thought this was pretty suitable as Q1, but many others thought it was way too easy (really?)

#### 3.5.2 Q2

Straightforward in a different sense: working backwards from the proof statement using equivalent conditions pretty much leads to a solution no matter what path you take (there are a hilarious 15 official solutions to this).

#### 3.5.3 Q3

A problem about structure, both the structure of the polynomial and the structure of the sequence. The thing to focus on (which came quite naturally) was how their structures mutually enforce each other. In particular I realised pretty fast that with the sequence being increasing, it gave me so much control (infinite to finite) that I was sure I could solve it. And it turns out if you try to prove increasing for n=2 it's quite easy, and generalising it wasn't particularly difficult either.

#### 3.5.4 Q4

I found this easier than Q1, there was only one possible thing to do (I mean, 3034??) and of course it works. Many thought this was a good Q4 choice, and I completely agree (except the fact that it is an A3, how??).

#### 3.5.5 Q5

I am pretty much completely alone in this opinion, but I feel the main difficulty is the number of rabbit holes one could fall into, and I fell into many. Induction on powers of 2 in particular is *extremely* tempting, especially given the substructure of the equality case. In the end I put aside inducting and arrived at the key observation. I've heard many complain about the fact that the key idea was basically DP, and I would agree if not for the fact that I still completely missed it.

#### 3.5.6 Q6

Pretty amazing example of how far "imagining the solution" can get you. Prove three circles are coaxial but they're not related? Find 2 points with equal power. The only points that make sense are the symmetric ones, I tried  $AA_2 \cap BB_2 \cap CC_2$  but that wasn't anywhere close to the radax, so I tried  $A_1A_2 \cap B_1B_2 \cap C_1C_2$  and it looked legit. From there working backwards to prove concyclicity was a simply trigo bash. 2 marks! Some people argue that this problem lacks any creativity, I can't say much having not solved it, but what I would say is that that dosen't make it a bad problem (I mean c'mon it's geom, not combi).

#### 3.6 July 10: Disneyland

(07:00) During breakfast, Po-Shen Loh came to our table to talk to us. He asked us about Q5 and revealed that one of the US contestants didn't solve it. Somewhere around this time Drew told me that Hong Kong had more solves for Q5 than Q2, which was just preposterous (and true). I also made the mistake of asking Prof Loh how expii was doing, and he launched into another passionate speech about how he is revolutionising the American education system (I wasn't against it, in fact I ardently support his initiative, its just that I've heard it all during last year's SMO prize presentation). Nicholas handed us Suica cards with 3000 Yen for us to spend for the day.

(08:00) We were running late for the train and had to jog to the station (which was a surprisingly common occurrence). The Japanese rush hour stories are real, I don't think I've come close to experiencing the worst of it, but it was hard enough navigating through a tsunami of people. Trains were pretty similar to Singapore's, except with cushioned seats. On the way there, we took a look at the marking schemes for the questions, which were sent to me by a helpful informant. I was pretty confident of a 77772. We arrived at Disneyland around 9 and endured the entrance queue in the summer sun - not very fun.



Figure 27: Disneyland!

Personally, Disneyland as an excursion was kind of mid, and the long snaking queue meant that I spent cumulatively a few hours playing crazyhouse with Qinxi. The rides also weren't particularly exciting or anything, but we took some nice photos. We had lunch with the Canada team and trolled them with ridiculous predicted scores.



Figure 28: What is it with IMO and theme parks?

(18:00) After taking the train back we met the rest of the Singapore delegation and went for dinner at some drinking place. At some point we got on the wrong train, and thanks to Yanxi we realised it before we were too late. On the journey back to Chiba, I was scrolling through the APMO threads on AoPS (at this point, the APMO results *still* weren't released), and I stumbled upon a post saying that CentroAmerican 18/6 was the pole polar dual of the Windmill problem, and the CentroAmerican problem was used by Wee Kean in one of our trainings. I was mind blown, and so (I assume) was the rest of the team and the observers.



Figure 29: Singapore team dinner.

We did not know at that point, but the observers were busy trying to fight for 2+1s on Q5 and spent dinner thinking about it. After dinner, the Singapore team went back to the Dragon Room and tried to learn a game called Figgie, invented by Jane Street to teach risk-taking or something like that. We tried

to entertain the guy but frankly it wasn't very fun. We did collect some free card decks from the room and played one game of air hockey, and it soon hit closing time. I slept well that night, having been on my feet the whole day.

#### 3.7 July 11: Excursions

(10:00) We woke up later than scheduled and decided to ditch breakfast for a cafe in central Tokyo. Funny story: Yanxi and I ordered hot milk tea, and because it comes without sugar we had to put our own sugar cubes. Qinxi, on the other hand, ordered the cold one, which comes with syrup, but as you can guess he put in the sugar cubes too, which led to a few minutes of us laughing at him.



Figure 30: Breakfast!

(12:00) We went to Ueno park, and honestly walking in midday under the sun is quite something. We rented three boats, which were irritatingly manual powered, and spent thirty minutes playing catching at 2km/h, and trying to smash into each other. I severely regret wearing a cotton shirt at this point, and this was the most exercise I've gotten since maybe Feburary.



Figure 31: Impressively well-positioned shot!

Exhausted, we went to eat at a Ramen place, which came with a crazy variety of sizes. I definitely chose a size too large, because I had no appetite to eat anymore while the rest had crepes for tea.



Figure 32: Small, medium, big, special, explosive, biggest

The rest of the day was filled with shopping, which I didn't particularly enjoy as well, but it was very fun to walk around foreign streets. I was pleasantly amused when Drew sent a sign which said HXGG (turns out it was HXCC and he edited it, but close enough).



Figure 33: LOL!

(17:00) We crossed the road and were greeted by 100,000 trees in the Meiji Jingu's forest, and this was my favourite part of the excursions. The peace of being surrounded by nature in the middle of the world's largest metropolis is ethereal.



Figure 34: One does not simply put a forest in here.

(19:00) At this point, the jury meeting should have ended. It barely started. On the ride back, we were busy scanning the scoreboards and I told Drew our top 15 was basically guaranteed. Singapore still had a Q5 column empty, for a good reason. My scores were basically fixed at what I expected, which was a pretty amazing feeling. When we went back to the hotel for dinner, the displays outside were cycling through all the scores, and there was quite a big crowd gathered outside.



Figure 35: Live scoreboard outside the dining hall.

India in particular did very well, and from what I gather big things were expected of this batch. Romania also cemented their their status at the top, after two consecutive years of top 5 after decades of 20ish. Mexico also saw a really big improvement this year. After a while, the SGP scores were completed, and both Wenhao and Yanxi got an extra 1 mark on Q5. (I will leave the full details for X-men night). I

went over to talk with Atul, and we congratulated each other for the 37. Celestia came over with the "I told you 4 months ago you'd break the 3/6 barrier" (he did, I forgot), and we received souvenirs from Philippines, Thailand and Bangladesh.

(22:00) Impatiently waiting for the results and the jury meeting to conclude, Drew, Yanxi, Edden and I decided to invite Archit to our rooms for pretty much completely no reason. It was there that I learnt the origin of "starchan" as his AoPS username (spoiler: it's epic). He tried to teach us fish (some card game) but unfortunately we were lacking a 6th player. So Edden left leaving the 4 of us and we decided to teach him Haidilao instead. Halfway through the game, the cutoffs were finally released like a dam breaking open - on MODS, on whatsapp, on AoPS, I even received several messages on Instagram: 18/25/32. Yanxi was extremely happy and we were all happy for him. This left the trickier issue of telling Qinxi and Wenhao about the cutoffs. The rest of the SGP delegation returned quickly and the observers were slightly disappointed that they couldn't dramatically reveal our Q5 scores to us; we already saw it during dinner.

(23:00) We gathered back in the observers' room for a final debrief, and funnily the first question that the observers asked was for us to order the scores of Iran, Singapore and Taiwan, who were consecutive in ranking and consecutive in scores. Edden crashed into the room really happily. Being so close to top 10 was a small disappointment but going from 24th to 11th (it later became 12th) is a massive improvement. Mr Zong told us some of the stories from the Jury meeting, including the Serbian contestant who continued writing for 20 minutes after the time has ended, which resulted in a deduction of 7 marks.

I can't remember much about that night, but it was spent partying on ONO 99, a really dumb game. The other X-men present were busy playing Hanabi, and Edden, Drew and I stayed to talk a bit about the contest itself. I tried to convince Drew why Q5 is hard - I don't think I succeeded. We noticed that the fridge in the hotel had a logo that looked suspiciously like Q5.



Figure 36: Were they hinting at it all along?

I got a bit tired by 3:30am (at that point the results *still* weren't on the IMO website - it turned out that the scores for 5 teams, most importantly Canada who finished in front of us, have not been recorded. Notably, Warren's **(CAN1)** score (40) was not included when deciding on the Gold cutoff, meaning instead of 53 gold medals as given during the voting process, it was in fact 54. How that happened I do not know, what happened to Japanese efficiency?)

#### 3.8 July 12: Closing Ceremony

(11:00) I woke up at 11 and spent a good 30 minutes trawling through the IMO website. The congratulatory messages started pouring in from the Singapore math community and I tried to reply to all (I was tired). More curiously my classmates also knew about my result and sent me messages too, I'm pretty impressed they not only remembered the date but figured out how to find the scores (love yall). Everybody else left hours before to go shopping at a nearby mall, but I FOMOed so I decided to take the 30 minute walk at midday to have lunch with the rest of the contestants. We had sushi, probably the last time we had Japanese food there. As we walked back, Xu Chen showed us a book that gave the equation for the perimeter of an ellipse as the QM of the two diameters.

(15:00) For the closing ceremony, I was seated near the front in the same row as Atul (we had a good time celebrating!). I talked a bit to Rogelio (MEX3) and Eric (USA4) who were seated beside me. The ceremony went by quite fast, with the names of HM awardees being displayed followed by the bronze medallists going up the stage, and then the silvers, and then the golds. Drew almost held the Singapore flag laterally inverted, which was a pretty funny scene. MODS collectively decided to orz Celestia, but due to a skill issue they did it prematurely (disappointing).



Figure 37: My batch of medallists, the second last one. Dude behind me is my homie the mayor.

At some point the Chairman of the IMO Board gave a speech, which included the phrase "many of you are disappointed, ..., and those at 41" at which point laughter erupted everywhere. I personally got my medal from the Mayor of Chiba, and was the only person to do so (weird flex). There were a total of 5 perfect scorers and they went onstage to collect a special prize, followed by the best female contestants from each continent.



Figure 38: Imagine no config error.

Afterwards, the IMO flag was transferred to the UK, and the 2024 video was screened - it had so much British humour I couldn't stop laughing (the Americans beside me were really confused).

(17:30) After the ceremony, the team went out to take group and individual photos, aptly in front of Hall 7. We then took photos with the Indian team, who gave us... some herbal soap and a stack of papers containing their TST (at which Atul pithily remarked: "they are all on AoPS"). I went to find Rafael to retake our wefie from last year, while the Australian team went around sneakily attaching small Koala softtoys onto everyone's blazers.

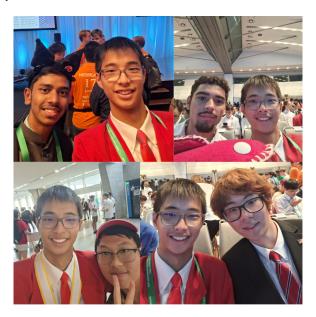


Figure 39: Bunch of selfies at the ceremony.

(18:00) We went into the dining hall for the farewell party, and this time there was meat (yes that's how low the bar was) and no chairs (pfft).



Figure 40: Live performance!

There was a whole band setup in the center and after eating a little I decided it's time to take currency exchange seriously. That night alone I spoke to probably around 50 countries and visited the hotel rooms of Luis (BOL3) and Iceland - the Iceland team also gave me some local biscuits which I didn't particularly like, so I gave it to Edden and Drew (I think they liked it, hopefully). I also spoke with the Kenyan team, who amazingly brought the currencies of all the countries neighbouring Kenya as well, so I got a 6 for 1 trade! At some point during the dinner Wee Kean decided to switch tags with me, and I proudly went to Chris with an "Observer B" tag. Not wanting to be outdone, Chris came back with an "Observer A" tag, so I of course went to Dr Soh and one-upped him with "Leader".



Figure 41: Instant kill!

(20:00) The band started playing a song about prime numbers which featured lyrics like "a safe Sophie Germain prime number. OK let's count 23. TWENTYTHREE" (in Japanese). There were about 300 people gathered in a circle around them trying to learn the associated dance, but honestly everyone was just waiting for the band to play 23 beats to run around the circle (musical chair vibes). I can't exactly remember but the song went up to 53 or 59, which is crazy.



Figure 42: Party Time!

The party formally ended there but informally it went on all night. As it turns out, Huawei gave all gold medallists a watch, and Edden and I (with Drew) went to the 46th floor to collect it.



Figure 43: Free Huawei watch on my bed.

There we met Enerelt **(MNG1)** and I joked with him about how his jacket, which said "Eenee" at the back, was a distraction during the competition because I couldn't believe anyone could have such a high proportion of 'e's in their name. I later followed Eenee and the Mongolian delegation back to the hotel and procured some Mongolian Tugrik. After that I spent quite a while at the Canadian's room talking to Kaixin and Warren, and much to my surprise Warren is only 15! (And has FCed APMO and basically IMO, what a legend.) Warren gave me 14 Canadian pins and I dumped them onto the SGP contestants, SIMON, and a random bunch of Koreans and Israelis (and I met Jalil **(AZE2)** for like the 100th time that week, and said goodbye to Bora). I met Franck **(CMR3)** in the lobby and got a bunch of Central African coins. I think Edden and Drew went to the arcade with the Thai team, I had no idea where anyone else was - Wenhao was in the room and I went to sleep after packing my bag.

#### 3.9 July 13: Departure

(08:00) Nothing much happened, we had breakfast, I said goodbye to a few people, and then we took one final photo with Nicholas and boarded the bus. On the way to the airport Xu Chen, Wee Kean and I spent some time devising the rules of continuous chess, which mostly went like:

- Every "piece" is a diameter 1 circle, no piece can intersect (tangent is fine).
- When capturing, the attacking piece's circle has to intersect the opposing piece's, but takes the position of the tangency right before.
- Rooks and bishops (and queens) naturally generalise, its just continuous now (stop before intersecting another piece). In fact, so does the king which moves anywhere in a radius 1 circle.
- Pawns move up to 2 units at the start, and up to 1 after that.
- Knights have some godly power, they can move to anywhere that is exactly  $\sqrt{5}$  from their current position, as long as it does not intersect a piece on the same side. Which means that knights can kill many pieces at the same time.

Of course there are a gazillion flaws, but who cares (and how does one even play this??).

(13:00) After shopping we decided to learn the rules of Shogi so we could entertain ourselves on the plane, but as it turns out it was too hard and I kept getting destroyed by the computer after 5 moves, which was depressing. I spent the rest of the time solving worms (withdrawal symptoms). We landed around 6pm local time, took a few final photos, and said our final goodbyes.



Figure 44: Our last photo as a team.

## 4 Acknowledgements

I would like to thank:

- The SGP team (Wenhao, Yanxi, Drew, Edden, Qinxi) and all the trainers (Aloysius, Xu Chen, Wee Kean, Shevonne, David) for all the beautiful memories forged, I'll forget all the math but remember all the clownery.
- Mr Zong, math olympiad teacher for the past 6 (!) years who forever changed my life. Thanks for always being there for us and all the best with future batches! (hopefully they're not as immature as me.)
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- My family (who will never see this), for their unwavering love and support.

"Enjoy this rare stage in your life where you can rightfully waste time on those cute math gems."

- XDitto



# Strange Equations Involving Functions 2022 Day 1

Problem 1. Find all functions  $f:\mathbb{R}\to\mathbb{R}$  such that any real numbers x and y satisfy

$$x^{3} + f(x)f(y) = f(f(x^{3}) + f(xy)).$$

**Problem 2.** Let  $2^{[n]}$  denote the set of subsets of  $[n]:=\{1,2,\ldots,n\}$ . Find all functions  $f:2^{[n]}\to 2^{[n]}$  which satisfy  $|A\cap f(B)|=|B\cap f(A)|$  for all subsets A and B of [n].

**Problem 3.** Find all functions  $f: \mathbb{N} \to \mathbb{N}$  such that any positive integers m and n satisfy

$$f^{f(m)}(n) \mid m+n+1.$$

**Information.** Each problem is worth 7 points. You have 4 hours and 30 minutes to solve the problems and write down your solutions. The use of calculators, engines and any other form of external help is forbidden. For other inquiries, contact gghx.

Submission Procedure. PM the following members on AoPS: Aritra12, EmilXM, gghx, oVlad and phoenixfire. Title your PM "[SEIF]" followed by your username. Please submit all your solutions under the same PM, do not create separate messages. You may submit one problem at a time. However, if you wish to submit solutions to the same problem again, please inform us clearly as the same problem.

Figure 45: Have fun with math olympiad while you can!